

Suddhendu Biswas and Vijay Kumar Sehgal

A Note on the Efficacy of the Abstinence Period for Preventing Reconception in the Context of the Period of Post-partum Amenorrhoea

1. Introduction

AN attempt is made here to analyze the efficacy of "abstinence" occasioned by the tradition of husband and wife staying separately following a child birth. The problem is to examine how far this practice prevents a reconception during post-natal period; especially when the lactational amenorrhoea or post-partum amenorrhoea already helps in preventing reconception.

The problem therefore, is to examine which of the two random variables viz., period of post-partum abstinence (X) and the period of amenorrhoea (Y) is stochastically larger. The data collected by International Institute of Population Sciences in respect of $P.P.A.$ as well as abstinence have been employed in this paper vide Karkal (1969), and Biswas (1973). The data provide marginal distribution of $P.P.A.$ and abstinence; but not the bivariate distribution of $P.P.A.$ and abstinence. Assuming that both the marginal distributions are log-normal and assuming a hypothetical (but plausible) correlation coefficient between X and Y , the joint distribution of X and Y is obtained; which enables us to obtain $P(X - Y > 0)$; obviously $P(Y - X > 0)$ implies the efficacy of abstinence period. The exercise is methodological. The same has been obtained by two different methods.

Post-partum abstinence which means a temporary separation of the spouses immediately following a childbirth (or reproductive wastage) on a voluntary basis or in keeping with various social customs and taboos, is an important demographic parameter which requires further examination in order to find out the efficacy of the same in averting future births (or conceptions) in the residual fertility span of the mothers. As a substantial part of the post-partum abstinence period (a.r.v. to be denoted by X) is characterised by the natural infecundability of the mother due to post-partum amenorrhoea (*P.P. A.*) following a child birth (a.r.v. to be denoted by Y). Therefore in order to evaluate the effectiveness of the post-partum abstinence in delaying the future date of the next conception, it is pertinent to obtain the probability distribution of $(X - Y)$, thereby enabling us to find out which of the two r.v.s. X or Y is stochastically larger. More precisely, what is the relative weight of $P(X - Y > 0)$ in relation to $P(X - Y < 0)$?

The present methodological exercise is an attempt to estimate the weight of $P(X - Y > 0)$ in relation to $P(X - Y < 0)$ vis-a-vis the joint bivariate probability distributions of X and Y .

Biswas (1973), made a comparative study of the post-partum amenorrhoea on the basis of the data collected in two independent surveys by International Institute of Population Sciences. The present study is based on the same data and its purpose to evolve a technique to obtain joint distribution of both the variables (X and Y) given only the marginal distribution of the same.

2. Material and Method

The basic data comprise the marginal distributions of the period of post-partum abstinence denoted by X and that of period of post-partum amenor-

TABLE 1—EMPIRICAL MARGINAL DISTRIBUTION OF THE MEAN PERIOD OF PPA-X (IN WEEKS)

<i>Mean Period of Abstinence (in weeks)</i>							
X	2.5	6.5	10.5	14.5	18.5	22.5	26.5
$Z = \log_e X$	0.9163	1.8718	2.3514	2.6741	2.9178	3.1135	3.2771
Frequency (fi)	30	168	164	108	104	100	58
X	30.5	34.5	38.5	42.5	46.5	50.5	65.5
$Z = \log_e X$	3.4177	3.5409	3.6506	3.7495	3.8394	3.9219	4.1821
Frequency (fi)	41	31	27	36	42	47	218

TABLE 2— EMPIRICAL MARGINAL PROBABILITY DISTRIBUTION O^
MEAN PERIOD OF PPA— Y (IN WEEKS)

<i>Mean Period of Post-Partum Amenorrhoea (in weeks)</i>							
<i>Y</i>	2	6	10	14	18	22	26
<i>W</i> - log _e <i>Y</i>	.6931	1.7917	2.3026	2.6391	2.8904	3.091	32581
Frequency (fi)	246	198	163	156	101	45	35
<i>Y</i>	30	34	38	42	46	52	64
<i>W</i> - log _e <i>Y</i>	3.4012	3.5264	3.6376	3.7377	3.8286	3.9512	4.1589
Frequency (fi)	31	35	34	32	31	32	35

SOURCE : Demography India (1973) Vol. II, No. 2, Page 201 (appendix) and page 206 Table 3.

rhoea denoted by *Y*. The empirical marginal distributions for the random-variables are given in Table 1 and Table 2 respectively.

The recorded data in respect of the frequencies for periods of *P.P.A.-X* and *P.P.A.-Y* differ. Therefore, at the outset it is desirable to obtain a methodology for the joint distribution of *X* and *Y*, based on the minimum common frequencies (i e. based on the records of 1174 months) on both. In the next place, the marginal distributions of *X* and *Y* are markedly skewed. Then assuming a series of hypothetical but plausible correlation coefficients together with the estimated mean and the variance from the data given by (3) and the joint probabilities of the different values of the r.v.'s *Z* (or the log-transformed *X*) and *W* (or the log-transformed *Y*) can be obtained which leads to generation of bivariate log-normal distribution of *X* and *Y*. The exercise in the line of above scheme has been performed to illustrate the methodology while taking a hypothetical value of the correlation coefficient to be $\rho = 0.5$. The results are given in Table 3.

Having obtained the joint probability distribution of the period of abstinence and that of the period of post-partum amenorrhoea, it was necessary, in the next stage, to employ the distribution so obtained to estimate the probability of the period of abstinence exceeding the period of post-partum amenorrhoea. In other words, our subject is to estimate $P(X - Y > 0)$ or the probability whether *X* is stochastically larger than *Y*. The complementary probability, viz $P(Y - X > 0)$

TABLE 3-JOINT DISTRIBUTION OF PERIOD OF ABSTINENCE AND PERIOD OF POST-PARTUM AMBNORRHOEA

		<i>Mean period of abstinence in weeks</i>								
δZ		<i>1-0986</i>	<i>0.5109</i>	<i>0.3365</i>	<i>0.2513</i>	<i>0.2006</i>	<i>0.1671</i>	<i>0.1431</i>	<i>0.1252</i>	
X†	$\log_e X \rightarrow Y$	<i>2</i>	<i>6</i>	<i>10</i>	<i>14</i>	<i>18</i>	<i>22</i>	<i>26</i>	<i>30</i>	<i>34</i>
	$\log_e y \rightarrow$	<i>.6931</i>	<i>1.7917</i>	<i>2.3026</i>	<i>2.6391</i>	<i>2.8904</i>	<i>3.091</i>	<i>3.2581</i>	<i>3.4012</i>	<i>3.5264</i>
2.5	0.9163	.37169	.217844	.16557	.10839	.01927	.06742	.05554	.04668	.07935
6.5	1.8718	.21043	.20552	.18428	.14851	.12809	.11192	.0989	.08823	.09283
10.5	2.3514	.111170	.14854	.14779	.13606	.12585	.11594	.10753	.0985	.09159
14.5	2.6741	.172980	.10335	.11083	.11236	.10927	.10508	.10054	.09598	.08405
18.5	2.9178	.036280	.0727	.08211	.0897	.00081	.00009	.08846	.08637	.07467
22.5	3.1135	.022344	.05128	.06128	.07109	.0743	.07568	.07585	.07552	.06531
26.5	3.2771	.014320	.03707	.04607	.05642	.06062	.06309	.06477	.06514	.05661
30.5	3.4177	.00948	.02726	.03508	.04497	.04949	.0525	.0545	.05581	.04827
34.5	3.5409	.00647	.02039	.02707	.03611	.04059	.03279	.04655	.04777	.04235
38.5	3.6506	.00452	.01547	.02113	.02921	.03346	.03666	.03900	.04094	.03662
42.5	3.7495	.00290	.0119	.01665	.02377	.0277	.03075	.03317	.03662	.03175
46.5	3.8394	.00260	.00926	.01326	.0195	.02308	.02596	.02829	.03145	
50.5	3.9219	.00460	.02063	.03084	.0476	.05847	.06732	.07482	.08674	
65.5	4.1821									

$\delta SZ - 1.7917/0.6931 - 1.0986$

†Mean period of post-partum amenorrhoea in weeks.

		$\delta_z =$	$\delta_z =$	$\delta_z =$	$\delta_z =$	$\delta_z =$	$\delta_z =$	$\delta_z =$	$\delta_z =$	$\delta_z =$
X	$Y \rightarrow$	38(.113	42(.091)	46(.0834)	50(.0769)	54(.0711)	58(.067)	62(.062)	66(.0595)	70
	$\log_e Y \rightarrow$	3.6376	3.7376	3.8286	3.9220	3.9889	4.06	4.27	4.189	4.2485
	$\log_e X \downarrow$									
2.5	0.9163	.03435	.03015	.0266	.02366	.0219	.01919	.01729	.01572	.01437
6.5	1.8718	.07185	.06545	.05996	.06519	.05096	.04726	.04396	.05103	.03839
10.5	2.3514	.86640	.08111	.07613	.07165	.06759	.05734	.06073	.05743	.05546
14.5	2.6741	.08741	.08347	.07079	.07634	0.06998	.06729	.06183	.06466	.06218
18.5	2.9178	.08164	.0791	.0767	.07444	.07217	.06998	.06786	.06584	.06391
22.5	3.1135	.07355	.07228	.08091	.07444	.06803	.06948	.06659	.06371	.06231
26.5	3.2771	.06210	.06468	.06407	.06948	.06256	.0617	.0608	.05988	.05894
30.5	3.4177	.05706	.05724	.0572	.05699	.06256	.05627	.0558	.05527	.05472
34.5	3.5409	.04981	.05038	.05074	.05699	.05094	.0512	.0501	.05077	.05028
38.5	3.6506	.04344	.04468	.04487	.03532	.04562	.04883	.04594	.04597	.04592
42.5	3.7495	.03784	.03882	.0396	.04001	.0407	.04107	.04135	.04155	.04169
46.5	3.8304	.03304	.03411	.03499	.03572	.03632	.0367	.03722	.03754	.0378
50.5	3.9219	.01480	.09559	.09815	.10224	.10493	.10404	.10934	.11112	.11268
65.5	4.1821									

is obtained first by considering different values of X lying between (2.5, 6.5), (6.5, 10.5), (10.5, 14.5), (14.5, 18.5)... (30.5, 63.5) weeks and obtain the probabilities of Y exceeding X for each of the classes (we ignore the case of $Y = X$ to avoid singularity in the distribution of X and Y ; as the two dimensional Lebesgue measure of the line $y = x$ is zero). These represent the conditional probabilities viz $P(Y > X | x < X < x + \delta x)$

$$(x, x + \delta x) - (2.5, 6.5), (6.5, 10.5), (10.5, 14.5), \dots$$

logarithmic transformation of both, however makes the distributions appear normal. In other words, while assuming the distribution of X and Y both to be log-normal, we proceed to estimate the bivariate distribution of X and Y given that $\log_e X = Z$ and $\log_e Y = W$ conform to a bivariate normal distribution. The mean and the standard deviation of $\log_e X = Z$ and $\log_e Y = W$ are obtained from the data as follows :

$$\left. \begin{aligned} \hat{\mu}_z &= \frac{3561.9805}{1174} = 3.034 \\ \hat{\sigma}_z^2 &= .7258, \quad \hat{\sigma}_z = .8519 \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} \hat{\mu}_w &= \frac{2720.174}{1174} = 2.317 \\ \hat{\sigma}_w^2 &= 1.1051, \quad \hat{\sigma}_w = 1.0512 \end{aligned} \right\} \quad (2)$$

Under the assumption that the joint distribution of (Z, W) is bivariate normal with parameter $(\mu_z, \mu_w, \sigma_z^2, \sigma_w^2, \rho)$, the regression of W on Z is linear and the conditional expectation of W given Z is univariate normal with mean and variance are as follows :

$$\left(\mu_w + \rho \frac{\sigma_w}{\sigma_z} (z - \mu_z), \sigma_z^2 (1 - \rho^2) \right) \quad (3)$$

$$\text{Then } \int_x P[Y > X | x \leq X \leq x + \Delta x] f(x) \Delta x = P[Y > X] \quad (4)$$

where $f(x)$ represents the marginal density function of X and

$$P[X > Y] = 1 - P[Y > X] \quad (5)$$

The details of calculation are summarised in Table 4.

TABLE 4—ESTIMATES OF CONDITIONAL PROBABILITIES OF PERIOD OF POST-PARTUM AMENORRHOEA (Y) EXCEEDING THE PERIOD OF POST-PARTUM ABSTINENCE (X) FOR DIFFERENT VALUES OF X

X (Period of post-partum abstinence)	Period of post-partum abstinence in logarithmic scale	Conditional probability of period of post-partum amenorrhoea exceeding the period of abstinence	
(1)	(3)	$P [Y > X x < X < x + \Delta x]$	$f(x) \Delta x$
2.5 — 6.5	0.9163 — 1.8718	0.3126	0.3426
6.5 — 10.5	1.8718 — 2.3514	0.23200	0.23254
10.5 — 14.5	2.3514 — 2.6741	0.177273	0.22998
14.5 — 18.5	2.6741 — 2.9178	0.139497	0.11755
18.5 — 22.5	2.9178 — 3.1135	0.100954	0.09114
22.5 — 26.5	3.1135 — 3.2771	0.078789	0.08518
26.5 — 30.5	3.2771 — 3.4177	0.060945	0.04940
30.5 — 34.5	3.4177 — 3.5409	0.04738	0.03492
34.5 — 38.5	3.5409 — 3.6506	0.036855	0.02641
38.5 — 42.5	3.6506 — 3.7495	0.028592	0.022993
42.5 — 46.5	3.7495 — 3.8394	0.021918	0.03067
46.5 — 50.5	3.8394 — 3.9219	0.016768	0.01022
50.5 — above	3.9219 — 4.1821	0.06197	0.025137

Hence from (4) we have $P(Y > X) = \sum_x (P(Y > X | x < X < x + \Delta x)) f(x) \Delta x$
 $= 0.2342366$
 $P(X > Y) = 1 - 0.2342366 = 0.7657634$ (6)

An alternative simple technique for testing $P(y > x)$ is given as follows :

$$\begin{aligned}
 P\{Y > X\} &= P\{\log_e Y > \log_e X\} \\
 &= P\{\log_e Y - \log_e X > 0\} \\
 &= P\{W - Z > 0\}
 \end{aligned}$$

$$\begin{aligned}
&= P \left[\frac{(W - Z) - (\mu_w - \mu_s)}{\sqrt{\sigma_w^2 + \sigma_s^2 - 2\rho\sigma_w\sigma_s}} > \frac{\mu_s - \mu_w}{\sqrt{\sigma_w^2\sigma_s^2 - 2\rho\sigma_w\sigma_s}} \right] \\
&= 1 - \Phi \left[\frac{\mu_s - \mu_w}{\sqrt{\sigma_w^2 + \sigma_s^2 - 2\rho\sigma_w\sigma_s}} \right]
\end{aligned}$$

where Φ is cumulative distribution of the standard normal distribution. In view of the data, the estimate of $P[Y > X]$ is given by

(for $\rho = -0.5$, known)

$$\begin{aligned}
&1 - \Phi \left[\frac{\hat{\mu}_s - \hat{\mu}_w}{\sqrt{\hat{\sigma}_w^2 + \hat{\sigma}_s^2 - \hat{\sigma}_w\hat{\sigma}_s}} \right] \\
&1 - \Phi \left[\frac{3.034 - 2.317}{\sqrt{.7258 + 1.1051 - .8955}} \right] \\
&1 - \Phi \left[\frac{.717}{.9672} \right] = 1 - \Phi(0.74) = 1 - 0.77 = 0.23
\end{aligned}$$

which is agreement with the earlier method.

3. Conclusion

Table 4 shows that abstinence period has a definite beneficial effect in delaying the date of next conception. Assuming correlation coefficient 0.5 between the period of abstinence and period of post-partum amenorrhoea it is shown that chance of abstinence period exceeding the naturally infecundable post-partum amenorrhoea is of the order of 77% approximately. This emphasises the imperative need of encouraging the practice of abstinence as a social custom helping family planning not only in averting future conception and births but also as a good measure for optimal spacing between two consecutive conceptions or births. With widespread urbanization and economic crisis, the practice of husbands and wives staying away from each other during the post-partum amenorrhoea is gradually becoming remote. The distribution of post-partum amenorrhoea is also changing with the change in the social set up of leading to reduction in the mean period of lactational amenorrhoea. In view of this newly emerging social change there is a greater need of emphasizing the custom of post-partum abstinence as one that positively benefit the family limitation programme.

References

1. Aitchison, J. and Brown J. A. C., 1957, *The Log-normal Distribution*, Cambridge University Press.
2. Biiwas, S., 1973, Abstinence, post-partum amenorrhoea and inter pregnancy interval, *Demography India*, 2 (2): 203-211.
3. Biswas, S., 1973, On the estimation of the mean waiting time period under fecundable and post-partum infecundable period from censored sample, *Inter-discipline*, 10 (3) : 43-53.
4. Cox, D. R., 1959, Analysis of exponentially distributed life times with two types of failure of R.S.S.B. 411-421.
5. Grundy, P. M., 1952 The fitting of grouped truncated and grouped censored normal distribution, *Biometrika*, 39 (384): 252-259.
6. Oupta, A. K., 1952, Estimation of the mean and standard deviation of a normal distribution, *Biometrika*, 39 (384) : 260-273.
7. Irwai, J. O., 1942, The distribution of the logarithm of survival times when the true low is exponential, *Journal of Hygem, Cambridge*, 42 : 328-333.
8. Karkal, M., 1969, *Post-partum Amenorrhoea in Greater Bombay*, A survey Report of the Demographic Training and Research Centre, Bombay, April 1969.
9. Karkal, M., 1969, *Abstinence after Delivery*, A Survey Report of the Demographic Training and Research Centre, Eombay, May 1969.